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LINEAR QUADRATIC REGULATOR FOR STRUCTURE UNDER ON-LINE PREDICTED FUTURE SEISMIC EXCITATION

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SUMMARY

A predictive-adaptive (PA) control algorithm has been developed for a structure under a seismic excitation. This algorithm analyses information of an observed seismic excitation, estimates future structural responses and determines the control force for the structure, based on the linear quadratic regulator. That is, at a given moment t_k: (1) seismic excitation information is converted to an autoregressive model, which forms the state equation for the excitation; (2) the identification model is combined with the structural model to build a state equation in an augmented space; (3) the weighted quadratic norm of the state vector and the future control force is formed as a cost function for estimating future responses; (4) the Ricatti equation is solved to find the optimum value of the cost function; and (5) the optimum gain matrix is obtained, and the control force is determined. The PA algorithm is not restricted to one type of control system, but can be applied to both an active driver system and an active tendon system. Its effectiveness is confirmed by numerical experiments for 1DOF and 3DOF structural models under sine and seismic excitations.

KEY WORDS: structural control; predictive-adaptive control; identification; feedforward control; seismic excitation; numerical experiment

INTRODUCTION

It is generally believed that feedforward (FF) control is more effective than feedback (FB) control. Thus, if we have advance knowledge of an excitation information to which a system is to be subjected, we can prepare the system to counter it. That is, the advance knowledge can be used to deduce an FF term which can counter the excitation. With the FF control, we could theoretically realize a system that is completely unaffected by the excitation if sufficient power were supplied. However, the power of control force is limited. In addition, a real system is exposed to unknown disturbances such as noise. To suppress the disturbances, the control algorithm must incorporate an FB term as well as the FF term. The instantaneous-optimization control[1] involves both terms, as it adds the instantaneous counteraction against the excitation to the FB term. The FF term in the instantaneous optimization balances the excitation at each moment. Since the excitation possesses dynamic characteristics, a cleverer control could be designed if this characteristic is involved in the algorithm as well as instantaneous balance. For a stationary excitation whose dynamic characteristics are known in advance, the FF control in this sense can be realized. The effect of the stationary state with FF–FB control has been confirmed.[2] However, it is impossible for a real structure to adopt it under a seismic excitation, since we cannot obtain the excitation information in advance.

The authors introduced an on-line method[3] for identifying excitations using known-in-advance information, which can predict near future information. By incorporating the predicted information into the Linear Quadratic Regulator (LQR), the authors have constructed a predictive-adaptive (PA) control algorithm. To introduce this algorithm, the equations of the structure and excitation are combined to construct one state

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equation in an augmented space. The solution of the Ricatti equation brings the optimal gain for the state vector in the augmented space. The obtained control force should reflect the future responses to the predicted excitation.

This report shows the results of numerical experiments and confirms the effectiveness of the proposed algorithm. Parametric studies on a 1DOF model find the appropriate dimensions for the identification and the duration for the cost function. Experiments using several weight matrices for the cost function compare the induced algorithm with the FB control. Experiments on a 3DOF structural model represent the basic performance of a multi-storey building.

AUGMENTED STATE SPACE

Structural model

The equation of motion at a moment t_r for an n-degree-of-freedom structure under a seismic excitation is

$$My''(t_r) + Cy'(t_r) + Ky(t_r) = -MVw(t_r) + Uu(t_r), \tag{1}$$

where M, C and K are $n * n$-dimensional matrices for mass, damping and stiffness, respectively. $y''(t_r)$, $y'(t_r)$ and $y(t_r)$ are n-dimensional vectors representing acceleration, velocity and displacement, respectively, for each degree of freedom. V indicates the positions of the degree of freedom on which the seismic excitation acts. $w(t_r)$ is a scalar for the acceleration of the excitation. U indicates the positions of the degree of freedom where the control force $u(t_r)$ acts. By setting an appropriate U as in Figure 1, the proposed algorithm can be applied to any control system which generates the control force with actuators, such as an active mass driver system or an active tendon system.[4–8]

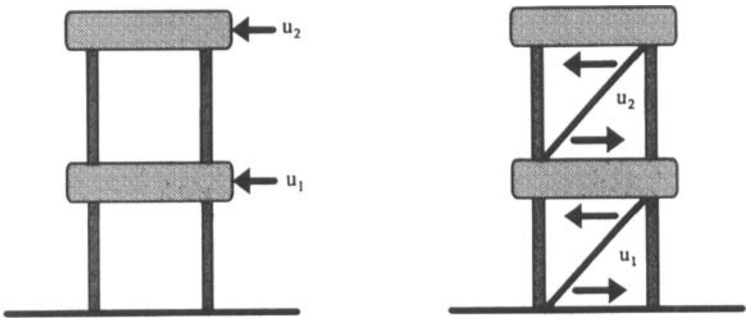
Equation (1) can be converted to the state equation for the sampling time Δt:

$$x(r+1) = Ax(r) + Dw(r) + Bu(r), \tag{2}$$

where

$$x(r) = \begin{Bmatrix} y'(r) \\ y(r) \end{Bmatrix}, \quad A = \exp(A_0\ \Delta t), \quad A_0 = \begin{bmatrix} -M^{-1}C & -M^{-1}K \\ \mathrm{I} & 0 \end{bmatrix}, \quad D = A_1 D_0, \quad B = A_1 B_0,$$

$$A_1 = \int_0^{\Delta t} \exp(A_0 \cdot \tau)\, \mathrm{d}\tau, \quad D_0 = \begin{bmatrix} -V \\ 0 \end{bmatrix}, \quad B_0 = \begin{bmatrix} M^{-1}U \\ 0 \end{bmatrix}.$$



$$\mathrm{Uu} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_1 \end{Bmatrix} \qquad \mathrm{Uu} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_1 \end{Bmatrix}$$

Control forces produced by Active Mass Driver **Control forces produced by Active Tendon**

Figure 1. Model of control system

Excitation model

At a given moment $t_k = k\,\Delta t$, assuming that a linear relationship holds with an error $e_k(r)$, and that a p-dimensional autoregressive (AR) model is adapted as a standard model in a linear space, the information of a seismic excitation $w(r)$ at other moments $t_r = r\,\Delta t$ can be expressed by

$$v(r+1) = E_k v(r) + He_k(r), \tag{3}$$

$$w(r) = F_k v(r) + L_k e_k(r), \tag{4}$$

where

$$E_k = \begin{bmatrix} 0 & 1 & \cdots & \cdot & 0 \\ 0 & 0 & 1 & \cdot & 0 \\ \vdots & \vdots & \vdots & \cdot & \vdots \\ 0 & 0 & \cdot & \cdot & 1 \\ -a_p(k) & -a_{p-1}(k) & \cdots & \cdot & -a_1(k) \end{bmatrix},$$

$$v(r) = \begin{Bmatrix} w(r-p) \\ w(r-p+1) \\ \vdots \\ w(r-2) \\ w(r-1) \end{Bmatrix}, \qquad H = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix},$$

$$F_k = -b_0[a_p(k)\ a_{p-1}(k) \ldots a_2(k)\ a_1(k)], \quad L_k = b_0(k).$$

The parameters at t_k are obtained by the identification of the information from $t_{k-m} = (k-m)\,\Delta t$ to $t_{k-1} = (k-1)\,\Delta t$ and are computed by the Maximum Entropy Method (MEM).[9,10]

Augmented state space

From equations (2)–(4), the equations of both the structure and the excitation are expressed in one equation in an augmented space:

$$\begin{Bmatrix} x(r+1) \\ v(r+1) \end{Bmatrix} = \begin{bmatrix} A & DF_k \\ 0 & E_k \end{bmatrix} \begin{Bmatrix} x(r) \\ v(r) \end{Bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \{u(r)\} + \begin{bmatrix} DL_k \\ H \end{bmatrix} e_k(r), \tag{5}$$

or

$$z(r+1) = \tilde{A}_k z(r) + \tilde{B} u(r) + \tilde{D}_k e_k(r), \tag{6}$$

where

$$z(r) = \begin{Bmatrix} x(r) \\ v(r) \end{Bmatrix}, \quad \tilde{A}_k = \begin{bmatrix} A & DF_k \\ 0 & E_k \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \tilde{D}_k = \begin{bmatrix} DL_k \\ H \end{bmatrix}.$$

LINEAR QUADRATIC REGULATOR

Cost function

The PA control algorithm is introduced, based on the Linear Quadratic Regulator (LQR). Expectation of the sum of the weighted quadratic norm of the structural state vector and the control force is defined as a cost

function for optimization, which estimates future responses:

$$J = E\left[\sum_{r=k}^{k+L} (x(r+1)^{\mathrm{T}} Q x(r+1) + u(r)^{\mathrm{T}} R u(r))\right]$$
$$= E\left[\sum_{r=k}^{k+L} (z(r+1)^{\mathrm{T}} \tilde{Q} z(r+1) + u(r)^{\mathrm{T}} R u(r))\right], \tag{7}$$

where $E[\ ]$ indicates the expectation and

$$\tilde{Q} = \begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix}.$$

Ricatti equation

The Ricatti equation induced by the defined cost function is

$$S_k(r+1) = \tilde{A}_k^{\mathrm{T}} S_k(r) \tilde{A}_k + \tilde{Q} - \tilde{A}_k^{\mathrm{T}} S_k(r) \tilde{B}(R + \tilde{B}^{\mathrm{T}} S_k(r) \tilde{B})^{-1} \tilde{B}^{\mathrm{T}} S_k(r) \tilde{A}_k \tag{8}$$

with the condition at the end:

$$S_k(r+L) = \tilde{Q}. \tag{9}$$

If the duration for computing the cost function L is long enough, the stationary solution is obtained by the eigenvalue problem of the Hamiltonian matrix:

$$H_k = \begin{bmatrix} \tilde{A}_k + \tilde{\mathrm{B}}\mathrm{R}^{-1}\tilde{B}^{\mathrm{T}}(\tilde{A}_k^{\mathrm{T}})^{-1}\tilde{Q} & -\tilde{\mathrm{B}}\mathrm{R}^{-1}\tilde{B}(\tilde{A}_k^{\mathrm{T}})^{-1} \\ -(\tilde{A}_k^{\mathrm{T}})^{-1}\tilde{Q} & (\tilde{A}_k^{\mathrm{T}})^{-1} \end{bmatrix}. \tag{10}$$

Gain matrix and control force

The gain matrix $G(k)$ for $z(k)$ and the control force $u(k)$ at t_k are calculated, using the solution of the Ricatti equation $S_k = S_k(k)$ as

$$u(k) = G(k)z(k) = -(R + \tilde{B}^{\mathrm{T}} S_k \tilde{B})^{-1} \tilde{B}^{\mathrm{T}} S_k \tilde{A}_k z(k). \tag{11}$$

Putting equation (5) into the above, we can express the control force as

$$u(k) = -(R + B^{\mathrm{T}} S_k^{11} B)^{-1} B^{\mathrm{T}} (S_k^{11} A_k x(k) + S_k^{11} D F_k v(k) + S_k^{12} E_k v(k))$$
$$= G_k^{\mathrm{FB}} x(k) + G_k^{\mathrm{IC}} v(k) + G_k^{\mathrm{FF}}(k)$$
$$= u^{\mathrm{FB}}(k) + u^{\mathrm{IC}}(k) + u^{\mathrm{FF}}(k), \tag{12}$$

where

$$S_k = \begin{bmatrix} S_k^{11} & S_k^{12} \\ S_k^{21} & S_k^{22} \end{bmatrix}.$$

These terms suggest the feedback (FB), instantaneous-counteraction (IC) and feedforward (FF) terms of the control force, respectively. It should be noted that S_k^{11} is equal to the solution of the Ricatti equation for the simple state-feedback control. Thus, the first term indicates the state-feedback control force. Since $F_k v(k) \approx w(k)$, and S_k^{11} is independent of the excitation t_k, we can call the second the IC term, which implies the instantaneous counteraction to the excitation. The third term is derived from the convolution effect of the excitation, so we acknowledge it as the FF term.

PREDICTIVE-ADAPTIVE CONTROL ALGORITHM

Control algorithm

At a given moment t_k, the control force of the PA control is computed as follows:

(1) Solve the identification problem for the excitation information $\{w_i: i = k - m \text{ to } k - 1\}$ to express it by an AR model, then get $\{a_i(k): i = 1 \text{ to } p\}$.
(2) Convert the AR model into the state equation form, combine it with the structural model and express both in an augmented space.
(3) Solve the Ricatti equation to get S_k and compute the gain matrices G_k^{FB}, G_k^{IC} and G_k^{FF}.
(4) Determine the control force $u(k)$.

PARAMETRIC STUDY FOR 1DOF MODEL

Numerical experiments were conducted to characterize the induced algorithm. The following 1DOF structural model was assumed for parametric studies: weight: $98{\cdot}0 * 9{\cdot}8$ (kN), spring: $3{\cdot}948 * 0{\cdot}098$ (kN/m), damping factor: 0·05, natural frequency: 1·0 Hz. Sine 1·0 Hz, El Centro and Hachinohe, as shown in Figure 2, were used as input waves. Let $R = \{1\}$. The following three studies were conducted:

Study A

To find an appropriate dimension of the AR model p and an adequate excitation duration for the identification m, cases with $\{p, m\} = \{1, 20\}$, $\{1, 100\}$ and $\{20, 100\}$ were conducted. The duration for computing the cost function L and the weight matrix for the state vector Q were fixed as infinite and diag$\{5, 0{\cdot}5\}$, respectively. The results for El Centro are shown in Figure 3. The maximum response acceleration and displacement are plotted versus the maximum control force. Moreover, the maximum values of each term of the control force are drawn for the maximum control force.

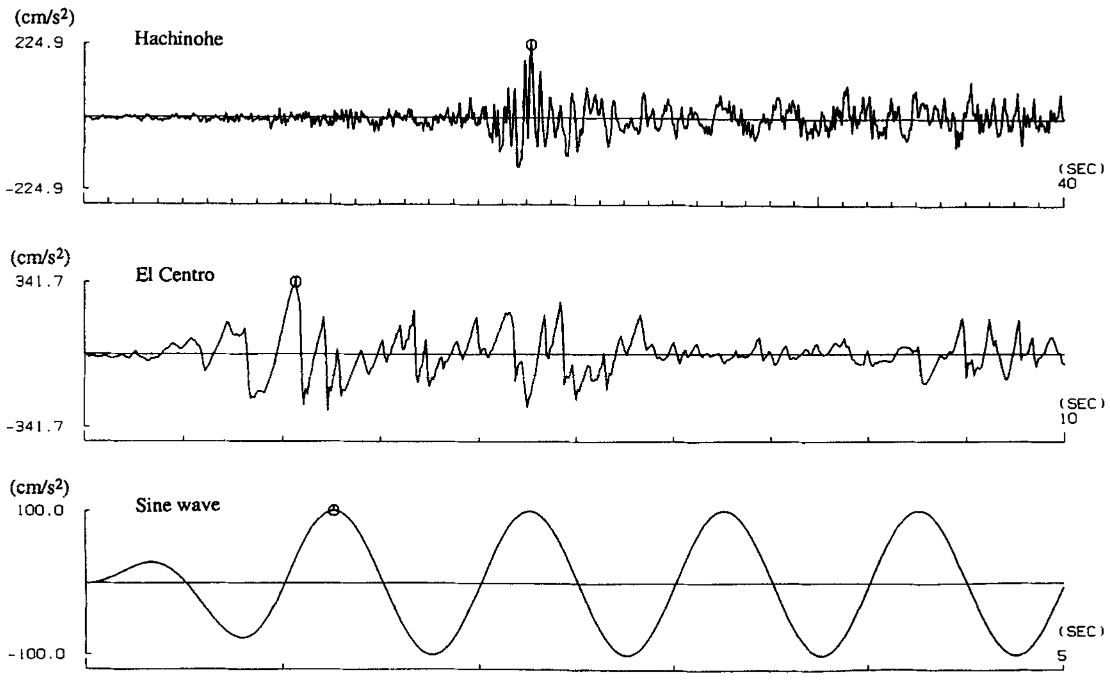


Figure 2. Input waves for numerical experiments

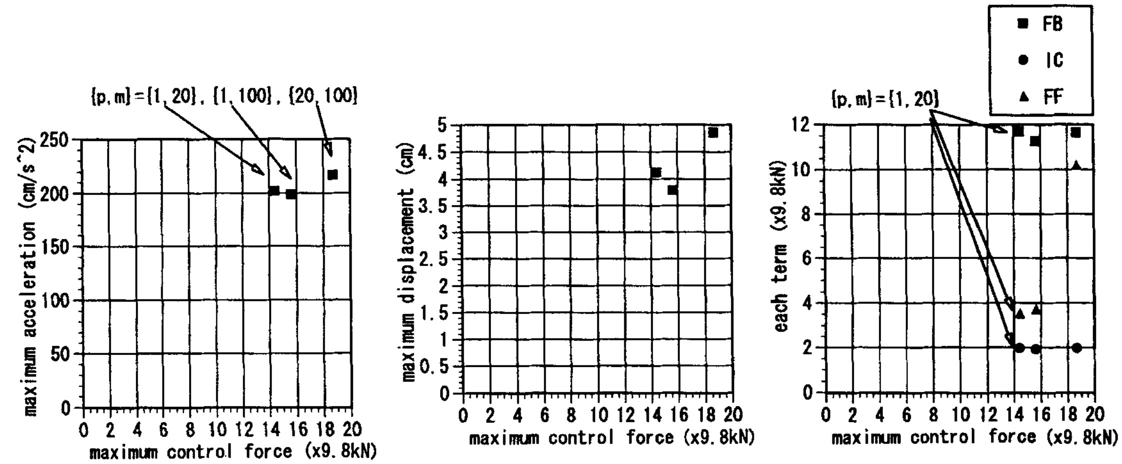


Figure 3. Difference due to identification model of excitation

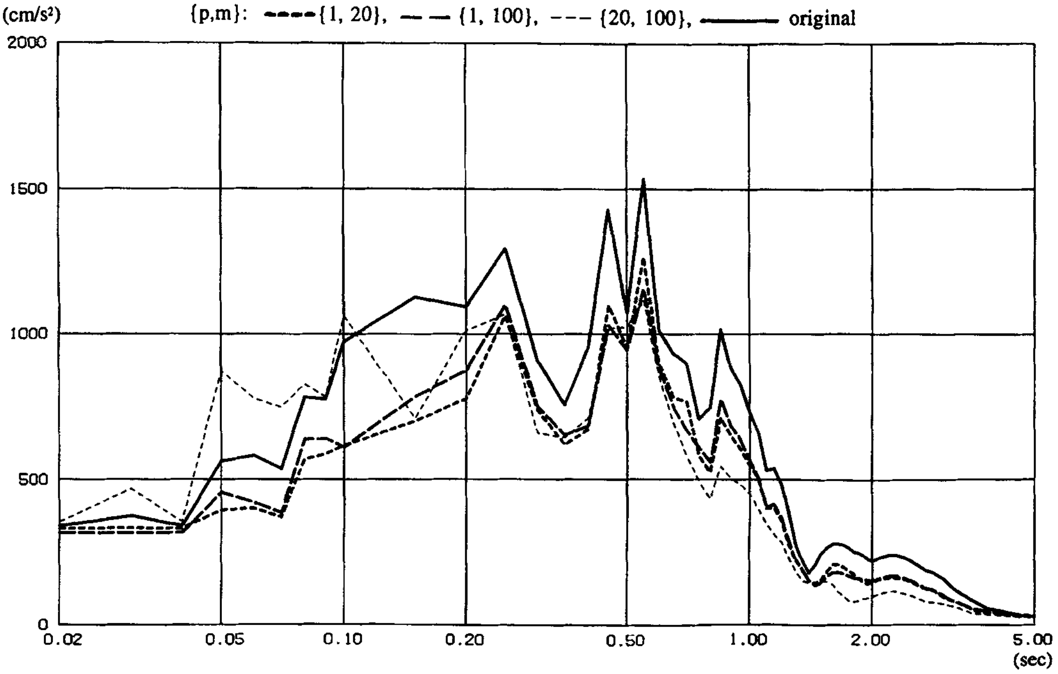


Figure 4. Acceleration response spectra of predicted 0·04-s-future excitation

The FB and IC terms were about in the same proportion in all cases. The difference was derived from the FF term. With a large-dimensional AR model, the control force became larger due to the large FF term, although it did not work well in suppressing responses. In fact, as shown in Figure 4 for the acceleration response spectra of the predicted 0·04-s-future excitation, high-frequency noise was picked up by a large-dimensional model. However, the control force by a small dimensional model was obtained from a filtered signal. Thus, it is judged that $p = 1$ is sufficient. Furthermore, when comparing the case for $m = 20$ with that for $m = 100$, we obtained small differences. For practical use, let $m = 20$ because of less computation. Thus, $\{p, m\} = \{1, 20\}$ was used in the following studies.

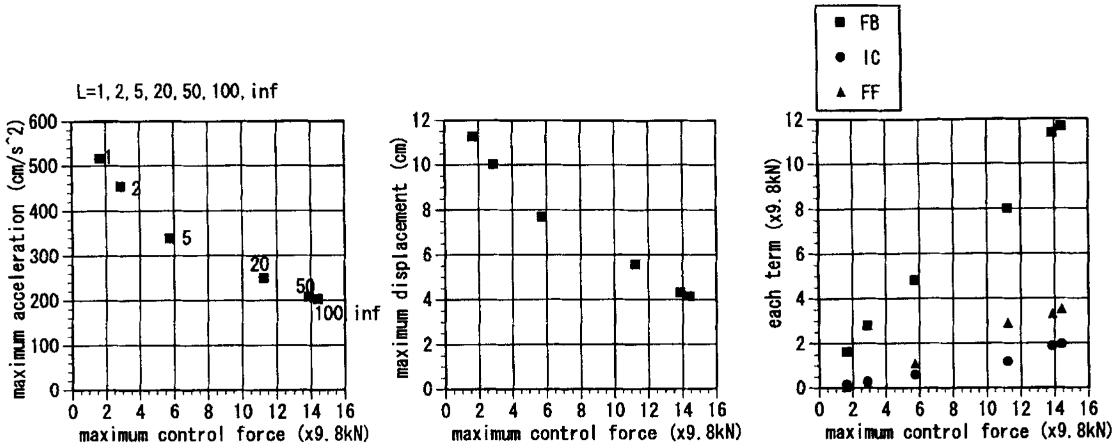


Figure 5. Differences due to duration of cost function

Study B

To confirm the difference due to the duration for computing the cost function L, the cases $L = 1, 2, 5, 20, 50, 100$ and infinite were conducted. The results for El Centro are shown in Figure 5.

The longer the estimated duration, the larger the obtained control force, although we could not distinguish the results of infinite steps and 100 steps in the figure. The larger the supplied control force, the greater the response suppression, although we can obtain a large control force by changing the weight of the norm. The largest difference among the cases was the influence of the FF term. If one or two steps were taken into account, the FF term was zero or too small. When a longer duration was estimated, a relatively large FF term was obtained, although the computation time became larger, except when estimating infinite duration, which was obtained by the eigenvalue problem and computed fast. Therefore, to consider the influence of the FF term, L was assumed to be infinite, that is, the stationary LQR was supposed in the following studies.

Study C

To compare the PA control with FB control, we conducted numerical experiments with various weights of norm. Cases were conducted with $\{Q_j\colon Q_j = q_j * \mathrm{diag}\{1, 10\}, q_j = 10, 5, 1, 0{\cdot}5, 0{\cdot}1 \text{ and } 0{\cdot}01\}$. The results are shown in Figures 6–11 for each input wave. The gain matrices for the FB control were obtained by the stationary LQR.

It was affirmed that the PA control suppressed structural responses more than the FB control. Furthermore, the figures for the control force energy show that the PA control required a smaller amount of energy, so it also worked more economically. As shown in figures for each term of control force, when the control force became large, the FF and IC terms of the control force surpassed the FB term. Thus, we understand that the PA control is active if the control force is large enough, compared to the inertia force on the structure.

NUMERICAL EXPERIMENTS FOR 3DOF STRUCTURE

Conditions

Assume a three-storey building modelled as three masses connected by shear springs as shown in Figure 12. This model was basically the same as that in Reference 3. Its first natural period of the model was 1·23 s. The damping factor $h = 0{\cdot}05$ for 1·0 s was estimated for all springs. In addition to the structure, active-tendon-type control systems were supposed to be installed between storeys and operated by the PA

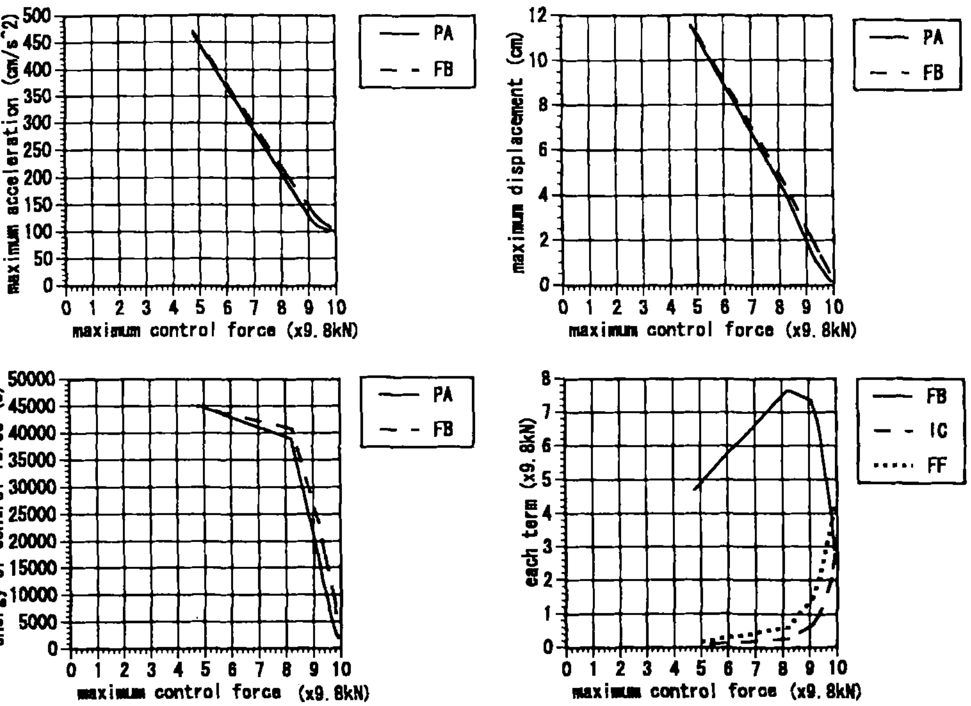


Figure 6. Responses to sine wave of 1DOF model

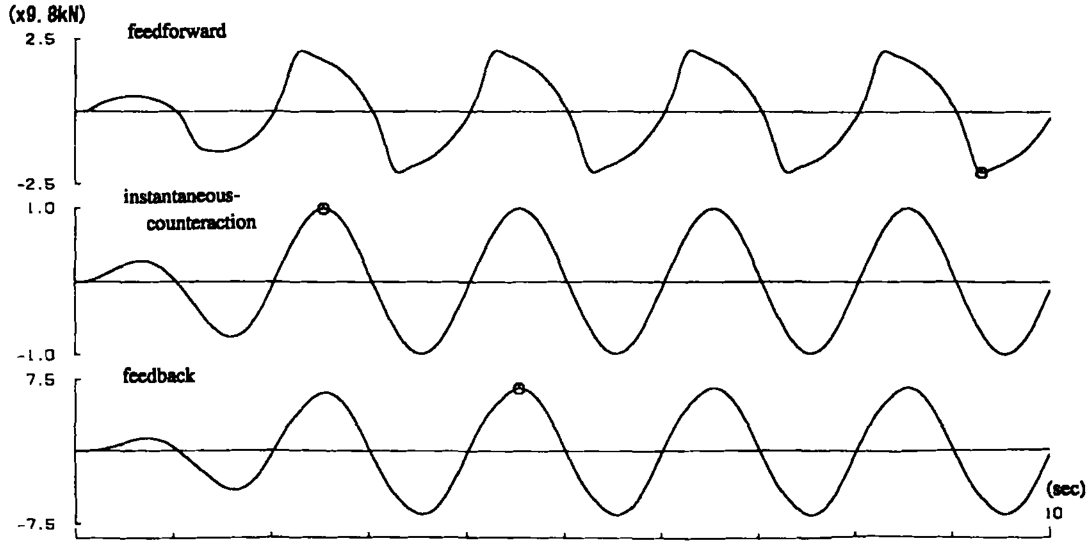


Figure 7. Control force terms of 1DOF model for sine wave

control. Numerical experiments were conducted for the sine wave ($f = 1/1{\cdot}23$Hz, maximum acceleration is 100 cm/s^2, input duration is 5·0 s) and El Centro as shown in Figure 2. Let $Q = \mathrm{diag}\{5, 5, 5, 50, 50, 50\}$ and $R = \mathrm{diag}\{2, 1, 1\}$ for the sine wave, and $Q = \mathrm{diag}\{0{\cdot}1, 0{\cdot}1, 0{\cdot}1, 1, 1, 1\}$ and $R = \mathrm{diag}\{2, 1, 1\}$ for El Centro. The results were compared with those of an uncontrolled structure.

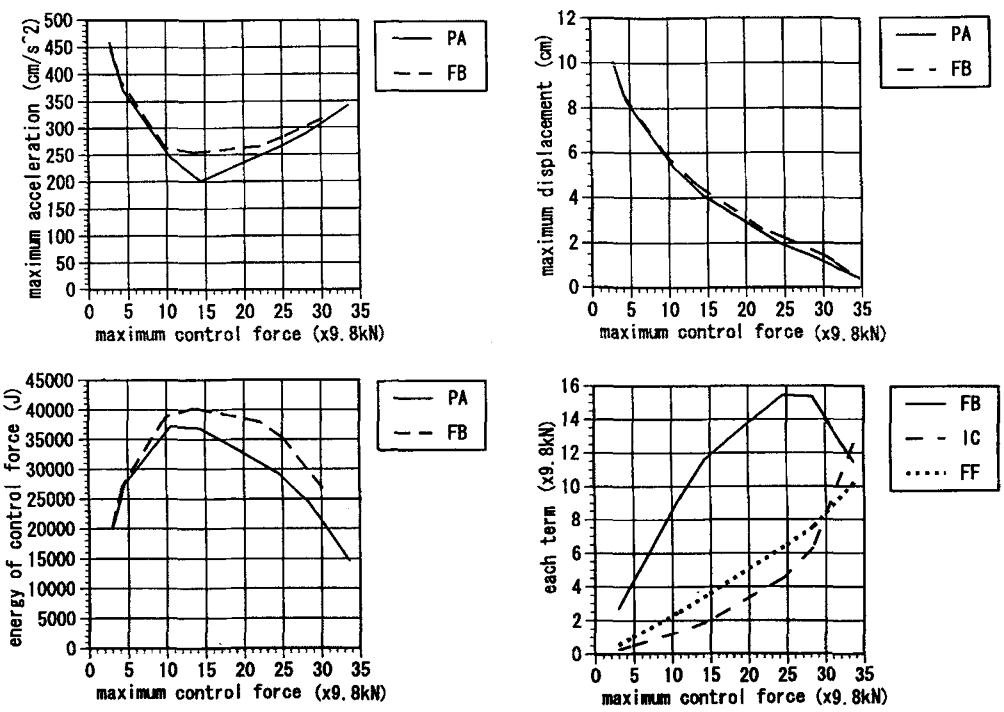


Figure 8. Responses to El Centro of 1DOF model

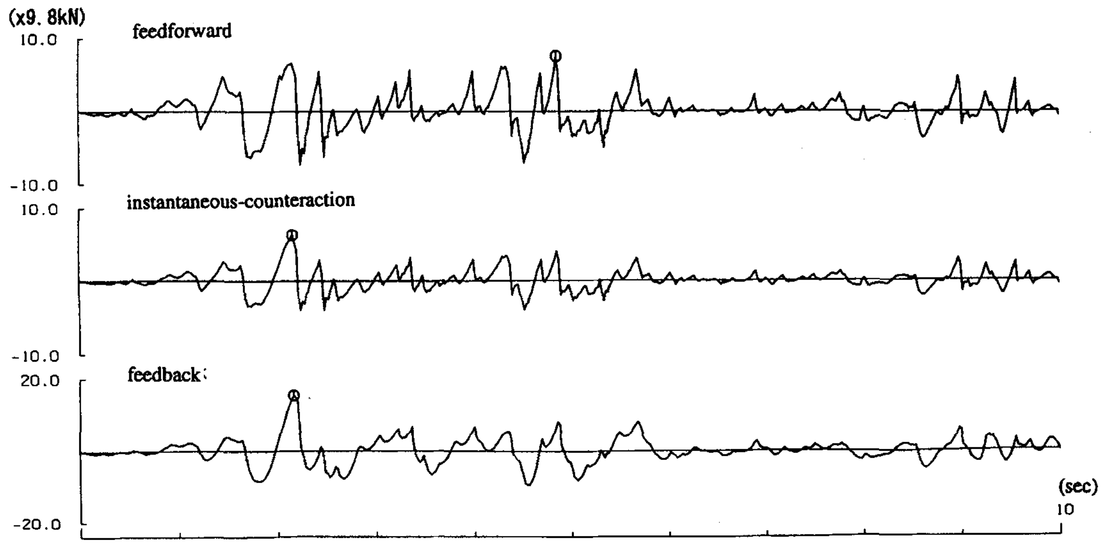


Figure 9. Control force terms of 1DOF model for El Centro

Figures 13 and 14 show the results for the sine wave, and Figures 15 and 16 show those for El Centro. The maximum responses for acceleration, displacement, control force and storey-shear force are shown in Figures 13 and 15. The time histories of each term of the control force are shown in Figures 14 and 16.

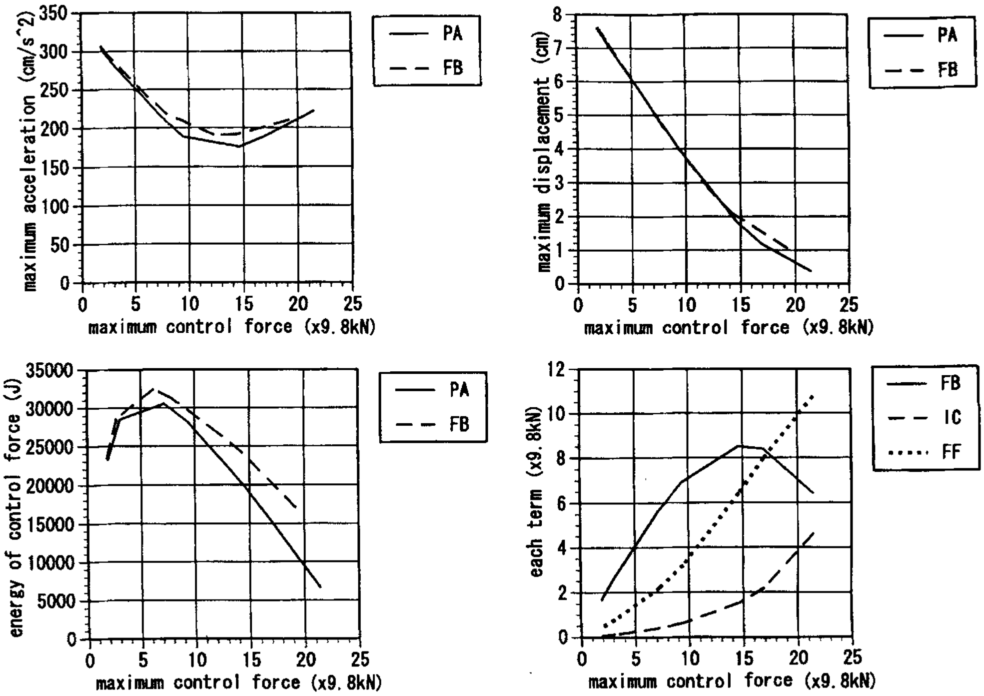


Figure 10. Responses to Hachinohe of 1DOF model

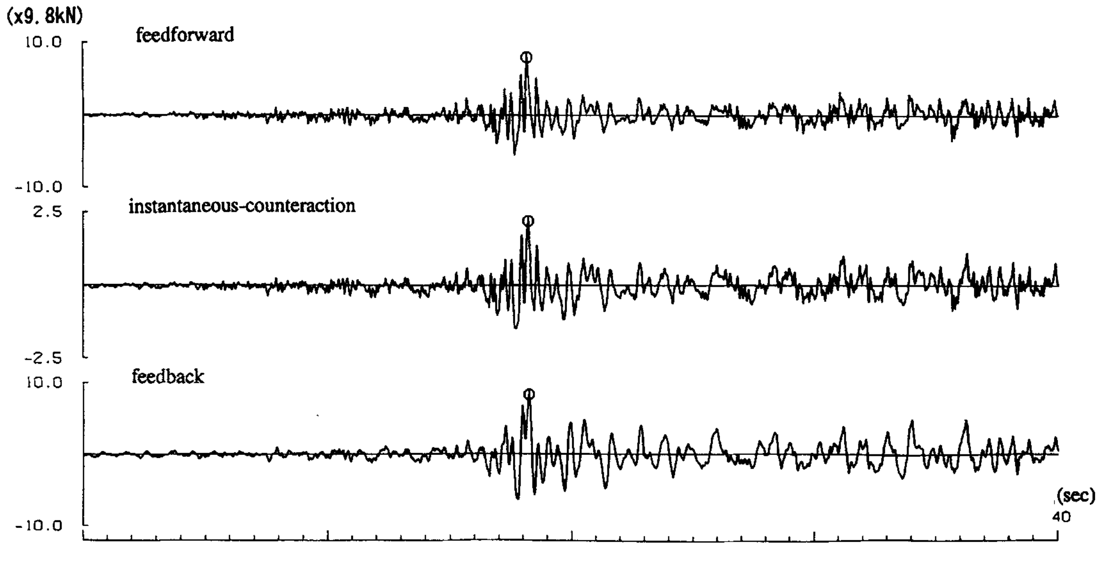


Figure 11. Control force terms of 1DOF model for Hachinohe

The sine wave responses were suppressed very well. Small control forces reduced the structural responses to the excitations. The time histories of each term show that the FF term came into play earlier than the FB term. This suggests that the PA control acted on the time lag during which the wave from the bottom was transmitted up through the structural springs.

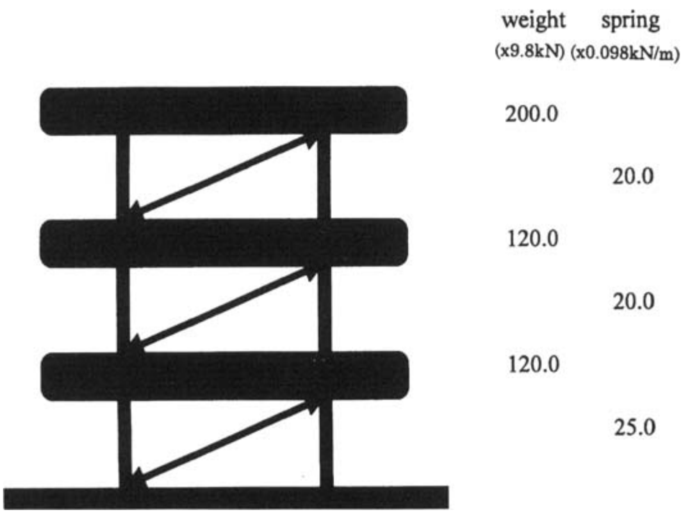


Figure 12. 3DOF structural model for numerical experiment

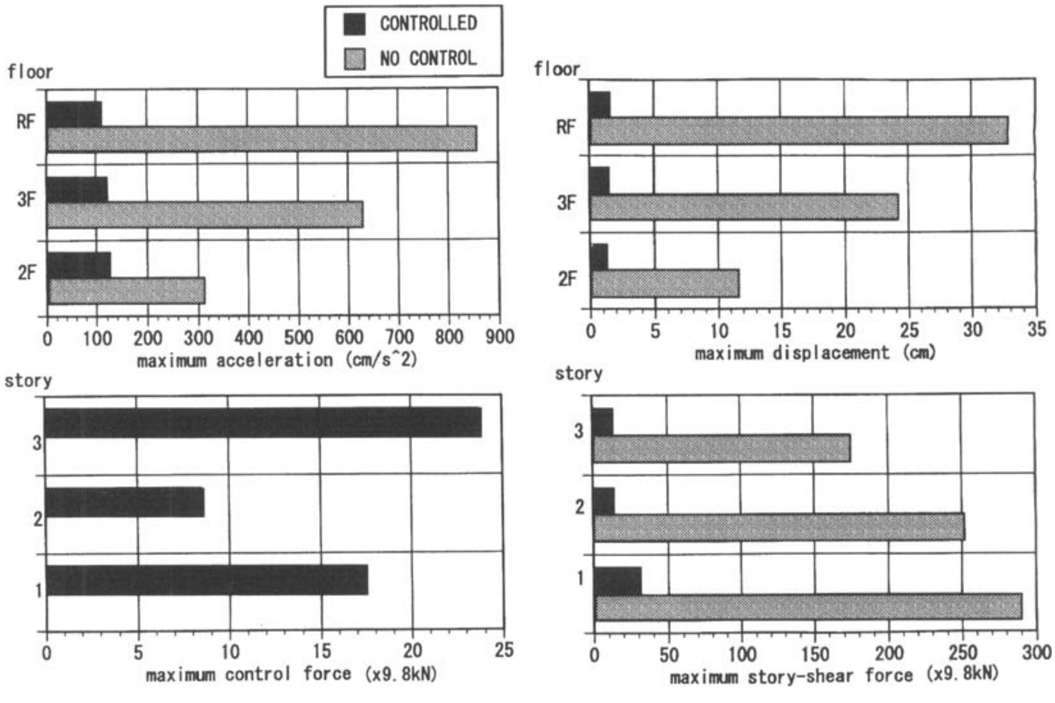


Figure 13. Responses to sine wave of 3DOF model

For El Centro, most of the structural responses were suppressed, although the maximum acceleration at the second floor with control was larger than that without control. This caused quick changes to the control force countering the transient excitation, as shown in Figure 16. It was difficult to suppress the acceleration due to the transient excitation, since the acceleration was not directly considered in the cost function.

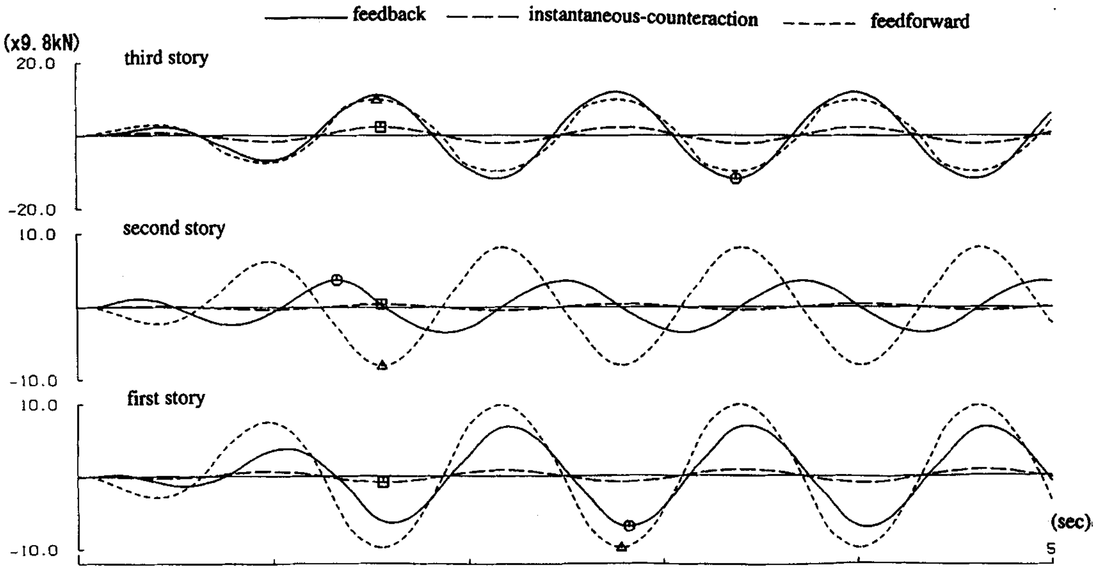


Figure 14. Control force terms of 3DOF model for sine wave

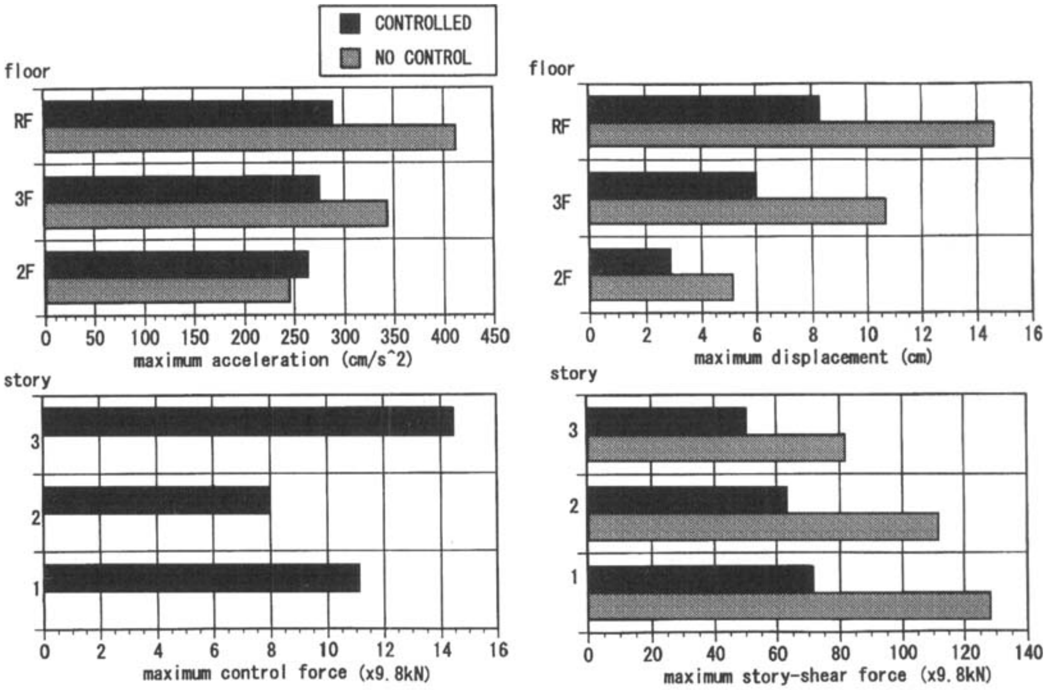


Figure 15. Responses to El Centro of 3DOF model

However, the large acceleration is not a problem for design unless it is excessive, and if other responses are suppressed. Thus, we understand that the PA control is effective.

A numerical analysis takes about 15 s for 10-s-duration El Centro, using a FORTRAN program on TITAN2-500X/HX, and including initial setting and response analysis. We judge that the PA algorithm can

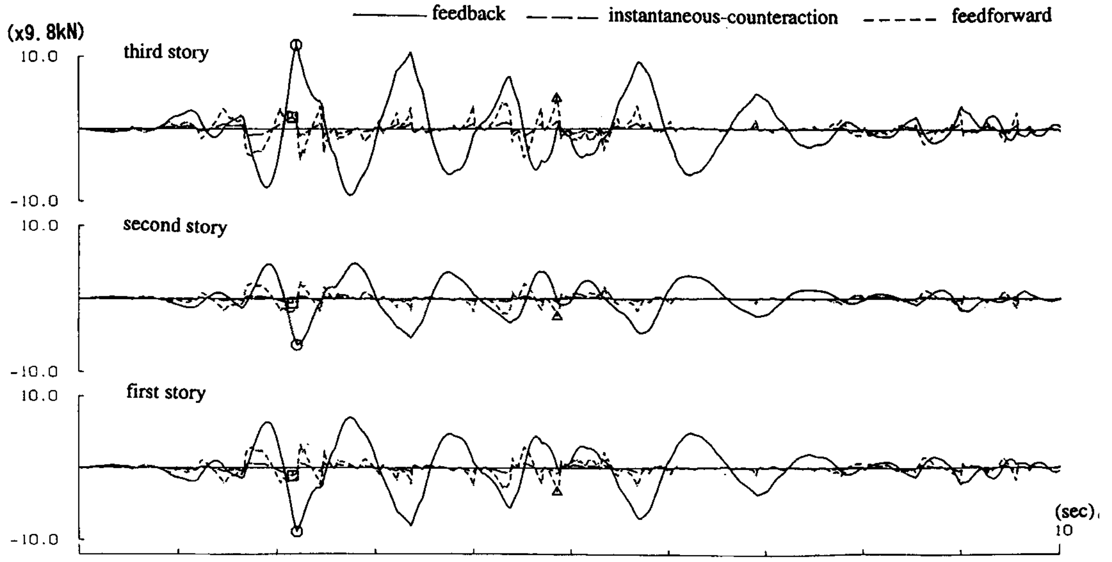


Figure 16. Control force terms of 3DOF model for El Centro

respond to real-time estimation. Therefore, we conclude that the proposed PA algorithm is practical as well as effective.

CONCLUSION

A predictive-adaptive (PA) control algorithm has been proposed for a structure under seismic excitations. To construct the algorithm, the state equation in an augmented space is constructed from a real-time identification model of an input excitation and a structural model. The optimal gain matrices are computed by the Ricatti equation, based on the Linear Quadratic Regulator for the augmented state equation. The obtained control force consists of feedback (FB), instantaneous-counteraction (IC) and feedforward (FF) terms.

Parametric studies on a 1DOF model confirmed that the PA control suppressed structural response more effectively than FB control. The FF term surpassed others if the control force is allowed to be large enough, compared with the inertia force on the structure. In other words, the PA control is especially useful for control systems which can produce large control forces.

Numerical experiments on a 3DOF structural model showed that the FF terms of the control forces at each storey acted to suppress the motion before it reached the point where the control system was installed. This confirms that the PA control efficiently suppresses responses against seismic excitations. The computational task of the numerical experiment was not much. Therefore, the proposed PA algorithm is practical as well as effective.

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